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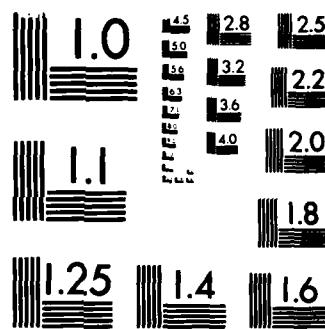
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Kenneth J. Arrow

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TECHNICAL INFORMATION, RETURNS TO SCALE,
AND THE EXISTENCE OF COMPETITIVE EQUILIBRIUM*

by

Kenneth J. Arrow

1. Introduction

'It is obvious that production requires information; in "standard" general competitive equilibrium theory, this information is embodied in the production possibility set. This kind of information may be called, "technical information;" it may be thought of as a recipe. The standard treatment is correct so long as information can be taken as given and not subject to alteration by deliberate choice. But in fact the acquisition of new technical information has proceeded on an increasingly rapid scale in the last century or more, and the volume of resources devoted to it has become very large. This fact has been much studied under the heading of the "economics of research and development."

In this paper, I want to concentrate on one particular aspect, the relation between research and development and the viability of competitive equilibrium. The interrelation between industrial organization and the demand for technical information has of course not gone unremarked. Most of the discussion has revolved about the thesis attributed to Schumpeter that perfect competition does not lead to optimal allocation of resources to innovation (Schumpeter [1939, Vol. 1, p. 107]). I want to examine here a slightly different, though related, point, that

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competitive equilibrium may not even exist when technical information can be acquired at a cost.

Speaking metaphorically, it can be said that production requires as inputs not merely the usual examples of labor, materials, and capital goods, but also the technical information for putting them together. But there is one important difference between technical information and other inputs. The latter will be in general increasing functions of the output. An identified body of conventional inputs, once used in production, cannot be used to produce further outputs. But information is not used up; the same knowledge can be used in producing output on any scale. In short, the technical information is a fixed cost of production.

Fixed costs of production of course create obstacles to the existence of competitive equilibrium. They are the barriers to entry in industrial organization theory. Indeed, that possession of technical information by incumbent firms is a barrier to entry by others was already pointed out by E. A. G. Robinson [1958, pp. 91-92]; see also Arrow [1974, pp. 38-39].

Wilson [1975] in an important paper pointed out the need for a more rigorous treatment. Since the very concept of information derives its meaning only in a context of uncertainty, the analogy to a fixed cost is too simplistic. The usual argument under certainty is that a price sufficient to cover average costs will exceed marginal costs; hence, under perfect competition there will be either zero or infinite output. But if the firm is risk averse under uncertainty, it will not

necessarily produce an infinite amount in any case. Further, information is not necessarily all or none; a firm may buy more or less information and obtain a corresponding reduction in uncertainty. Wilson showed, by example that even for a risk-averse firm, it was possible to find a sequence of decisions involving successively larger outputs and corresponding larger amounts of information which were successively better for the firm. This result tended to support the view that the possibility of acquiring information is indeed incompatible with competitive equilibrium.

In this paper, I wish to reexamine this question more explicitly with a simple model. In this, I allow in effect the output price to respond explicitly so as to achieve equilibrium if possible. I find a sufficient condition on the mean return (which is a reflection of the price) so that the firm's optimal choice of information and scale is unbounded. From this, it will be seen that non-existence of equilibrium is a very likely possibility under a number of circumstances. On the other hand, an example is given in which competitive equilibrium does exist.

As a preliminary, I give a brief account of the situation with risk neutral firms. Here, competitive equilibrium is essentially never viable if information will lead to higher expected returns.

It should be made clear that the assumptions on the relation between information and product made in this paper differ as between the risk neutral and risk averse cases. In the latter, increasing information reduces uncertainty, as measured by variance, but does not increase

mean performance. Since we cannot expect mean productivity to increase beyond limit as information increases, the results should give a correct implication for the possibility of unbounded optimal behavior.

2. The Risk Neutral Firm

To present the simplest possible case, we assume that the production process has one physical input and one output. The returns to scale are constant. The unit return, X , depends on an operating parameter, to be chosen by the firm, and on other factors not under its control, which we may term, as usual, the state of the world. The firm also has choice of a signal, S , from a set, Σ . However, there is a cost, $\gamma(S)$, if the signal S is chosen. (The notation is ambiguous; " S " will stand both for the designation of the signal and for the signal considered as a random variable. Thus $\gamma(S)$ is the cost considered as a function of the designation of the signal, not as a random variable depending on the outcome of S . The ambiguity will not create any difficulties.) Finally, the firm also chooses the scale of operations, a.

Given the choice of a signal, S , and a realization of that signal, the firm will make a choice of operating parameter, which will define the distribution of the unit return, X . For a given choice of signal, S , let v_S be the expected unit cost, the expectation being taken over both the realization of S and the other random factors affecting X for given choice of the operating parameter. Then,

$$\text{expected profit} = av_S - \gamma(S) .$$

If there exists a signal S for which $v_S > 0$, then the firm can achieve infinite expected profit by choice of an infinite scale, a . This is clearly incompatible with competitive equilibrium.

Note that the unit return depends on market price as well as on costs. Assume the market price known to the firm, so it is only seeking information on costs. Write c_S for the expected unit cost when the signal S is used to determine the operating parameter, so that,

$$v_S = p - c_S .$$

Then, in seeking competitive equilibrium, we can assume that p falls to prevent the optimality of an unbounded scale. To put it another way, we can expect that v_S will adjust. Hence, a necessary condition for competitive equilibrium is that $v_S \leq 0$, for all signals S . But in that case, for any signal, the firm can achieve zero profits gross of information costs, by setting $a = 0$, but no positive gross profits. The optimal policy for the firm is then to choose S so as to minimize $\gamma(S)$, say at \underline{S} . We can reasonably assume that $\gamma(\underline{S}) = 0$, i.e., it is possible to buy no information beyond that initially available to the firm. Then the only possible equilibrium price would be $c_{\underline{S}}$, if demand at that price is positive. But if there is any signal that is informative, in the sense that $c_S < c_{\underline{S}}$, it would be profitable at the competitive price $c_{\underline{S}}$ for any firm to purchase signal S , no matter how expensive, and then plan to produce at a sufficiently high scale to make a positive profit.

Therefore, if there is any signal for which expected unit cost is tive equilibrium when firms are risk neutral.

3. Risk Aversion with Variance-Reducing Information

In seeking to show that competitive equilibrium does not exist, a standard procedure is to show that supply will be infinite at some prices and zero at all others. If demand is positive at the boundary between the two sets of prices, then clearly non-existence will be established. In this case, increasing the scale of operations indefinitely will lead to indefinitely increasing uncertainty of total profits if the variance of the unit return is bounded away from zero. It can easily be seen that if the unit return has a non-degenerate distribution, and if the marginal utility of income approaches zero as income approaches infinity, then the optimal scale must be finite. Let

Y = income ,
X = unit return ,
a = scale of operations ,
and $U(Y) =$ utility of income .

Theorem 1. If $U(Y)$ is concave, with $U'(+\infty) = 0$, $Y = aX$, and X has a non-degenerate distribution, with a positive probability of loss, i.e., $P(X < 0) > 0$, then the optimal choice of a to maximize $E[U(aX)]$ is finite.

Proof: Let $W(a) = E[U(aX)]$. Assume that the distribution of $W'(a) = E[U'(aX)X]$.

If $E(X) < 0$, then $W'(0) < 0$, so that $a = 0$ is optimal. Now suppose that $E(X) > 0$. It suffices to show that $W'(a) < 0$ for a sufficiently large. Choose $\underline{X} < 0$ so that $P(X < \underline{X}) > 0$. Then,

$$E(X) = \int_{-\infty}^{\underline{X}} x f(x) dx + \int_{\underline{X}}^{\bar{X}} x f(x) dx > 0 ,$$

where $f(x)$ is the density of X . Since $\underline{X} < 0$,

$$(3.1) \quad \int_{-\infty}^{\underline{X}} x f(x) dx < 0 ,$$

so that,

$$\int_{\underline{X}}^{\bar{X}} x f(x) dx > 0 .$$

Since,

$$\int_{\underline{X}}^0 x f(x) dx < 0 ,$$

it follows by continuity that we can find $\bar{X} > 0$ such that,

$$(3.2) \quad \int_{\underline{X}}^{\bar{X}} x f(x) dx = 0 .$$

For $x < \underline{X}$, $ax < a\underline{X}$, so that $U'(ax) > U'(a\underline{X})$, and therefore, since $x < 0$,

$$U'(ax)x < U'(a\underline{X})x \quad \text{if } x < \underline{X} .$$

Similarly,

$$U'(ax)x < U'(a\bar{X})x \quad \text{for } x > \bar{X} .$$

For any $x < 0$, $ax < 0$, so that $U'(ax) > U'(0)$, and therefore,

$$U'(ax)x < U'(0)x ,$$

and the same holds if $x > 0$. Therefore, for any a ,

$$\begin{aligned} W'(a) &= \int_{-\underline{X}}^{\underline{X}} U'(ax)x f(x) dx + \int_{\underline{X}}^{\bar{X}} U'(ax)x f(x) dx + \int_{\bar{X}}^{\bar{X}} U'(ax)x f(x) dx \\ &< U'(a\underline{X}) \int_{-\underline{X}}^{\underline{X}} x f(x) dx + U'(0) \int_{\underline{X}}^{\bar{X}} x f(x) dx + U'(a\bar{X}) \int_{\bar{X}}^{\bar{X}} x f(x) dx . \end{aligned}$$

The second term on the right is zero, by (3.2). Since $\bar{X} > 0$, $U'(a\bar{X})$ approaches $U'(+\infty) = 0$ as a approaches infinity. Since $\underline{X} < 0$, $U'(a\underline{X})$ approaches $U'(-\infty)$, which is positive (possibly infinite), as a approaches infinity. From (3.1) it follows the right-hand side approaches a negative limit. Hence, $W'(a) < 0$ for a sufficiently large, which proves the theorem.

It follows from Theorem 1 that if uncertainty of the unit return is bounded away from zero, then an infinite supply is possible only if the mean return can be made arbitrarily large. But unit cost cannot be less than zero, and therefore the mean return is necessarily bounded by the price, taken as given by the firm. Therefore, infinite scales of operation can occur only if uncertainty is reduced to zero by purchase of information.

To bring this feature out sharply, I will postulate that mean return is a constant, uninfluenced by either the signal purchased or by

its realization. At present, I will assume this constant mean to be positive. What information does do is to reduce uncertainty. I will therefore measure the amount of information by the posterior precision of X , that is, the reciprocal of the variance of X conditional upon the signal (it will be assumed independent of the realization of the signal, as in linear normal regression). Let ,

$$v = E(X) ,$$

$$S_h = \text{signal indexed by } h .$$

Then, the assumptions are that,

$$(3.3) \quad E(X|S_h) = v > 0 \quad \text{for all } h \text{ and all realizations of } S_h ,$$

$$(3.4) \quad \sigma_{X|S_h}^2 = h^{-1} \quad \text{for all realizations of } S_h .$$

Let,

$$(3.5) \quad Z_h = h^{1/2}(X - v) ,$$

so that, from (3.3) and (3.4),

$$E(Z_h|S_h) = 0, \quad \sigma_{Z_h|S_h}^2 = 1 \quad \text{for all } h \text{ and all realizations .}$$

To further simplify, assume that Z_h has a constant conditional distribution, independent of h and of the realization S_h . Then, we can drop the subscript h , and write, from (3.5),

$$(3.6) \quad X = v + h^{-1/2} Z ,$$

$$(3.7) \quad Z \text{ is a random variable, } E(Z) = 0, \sigma_Z^2 = 1.$$

Finally, we have to assign a cost function to the signals. Following the usual analysis in Bayesian-normal sampling, we assume that the cost is proportional to h :

$$(3.8) \quad \gamma(s_h) = ch.$$

The realized income, Y , depends on the realization of the random variable, X , or, equivalently, Z , and on the choices of the decision variables, a (scale) and h (amount of information):

$$(3.9) \quad Y = aX - ch = av + ah^{-1/2}Z - ch = Y(a, h, Z),$$

from (3.6).

The firm seeks to maximize $E[U(Y(a, h, Z))]$ with respect to a and h . Suppose such a maximum exists and is interior. The first-order conditions are,

$$E[U'(Y) (\partial Y / \partial a)] = 0,$$

$$E[U'(Y) (\partial Y / \partial h)] = 0.$$

From (3.9),

$$\partial Y / \partial a = v + h^{-1/2}Z,$$

$$\partial Y / \partial h = -(1/2)h^{-3/2}Z - c.$$

Making the appropriate substitutions, we find,

$$v E[U'(Y)] = -h^{-1/2} E[U'(Y)Z] ,$$

$$c E[U'(Y)] = -(1/2) ah^{-3/2} E[U'(Y)Z] .$$

If we divide the first of these equations into the second, we find,

$$(3.10) \quad a = (2c/v) h = a(h) , \text{ say} .$$

This relation must hold at any optimal point.

Theorem 2. Let S_h be a one-dimensional family of signals about the unit return, X , where the cost of the signal S_h is ch . Suppose there is a random variable, Z , with mean 0 and variance 1, such that the distribution of X given S_h is the same as that of $v + h^{-1/2} Z$, where $v > 0$. If the firm has a finite interior optimal choice of scale, a , and signal, S_h , it must be that $a = (2c/v) h$.

I will tentatively assume that a and h are related as in Theorem 2 and ask when it will be optimal to have an infinite scale and infinite information demand, with the two in the ratio given by (3.10). Let,

$$(3.11) \quad Y(h, Z) = Y[a(h), h, Z] = ch + (2c/v) h^{1/2} Z .$$

The first and in itself interesting step will be to show that as h approaches infinity, $Y(h, Z)$ converges in probability to an infinite income; that is, the probability that $Y(h, Z)$ exceeds any given value of income can be made arbitrarily close to 1 by choosing h sufficiently large.

For any income level, y , define $z^0(h, y)$ so that,

$$(3.12) \quad Y[h, z^0(h, y)] = y .$$

Then $Y(h, Z) > y$ if and only if $Z > z^0(y, h)$. Solve for z^0 from (3.11) and (3.12).

$$z^0(h, y) = (v/2c) y h^{-1/2} - (v/2) h^{1/2} \rightarrow -\infty ,$$

as $h \rightarrow +\infty$. Therefore,

$$P[Y(h, Z) > y] = P[Z > z^0(h, y)] \rightarrow 1 \text{ as } h \text{ approaches } +\infty ,$$

for any y .

Theorem 3. Suppose the hypotheses of Theorem 2 hold. Then, by choosing a sequence of signals, indexed by h_n , with h_n approaching infinity, and, for each n , a scale $a_n = (2c/v) h_n$, the probability that the firm's income exceeds any prescribed limit approaches 1.

This theorem does not directly address the optimality of the unbounded policy. Since both a_n and h_n are approaching infinity, there are possibilities of large losses, even though the probabilities of them are becoming small. We have to look more directly at expected utility. Let,

$$\phi(z) = \text{density of } Z.$$

$$\begin{aligned} E\{U[Y(h, Z)]\} &= \int_{z^0(h, y)}^{+\infty} U[Y(h, z)] \phi(z) dz \\ &+ \int_{-\infty}^{z^0(h, y)} U[Y(h, z)] \phi(z) dz, \end{aligned}$$

for some arbitrary y . For $z > z^0(h, y)$, $Y(h, z) > y$, and, if U is monotone increasing in Y , $U[Y(h, z)] > U(y)$. Therefore,

$$(3.13) \quad E\{U[Y(h, Z)]\} > U(y) P[Y(h, Z) > y] + \int_{-\infty}^{z^0(h, y)} U[Y(h, z)] \phi(z) dz .$$

Suppose the second term approaches zero as h approaches infinity. From Theorem 3, the first term approaches $U(y)$. Hence, by choosing h sufficiently large, we can make $E\{U[Y(h, Z)]\} > U(y) - \epsilon$ for any y and any $\epsilon > 0$. For any y' , choose $y > y'$, $\epsilon = U(y) - U(y')$; then we can say that, for h sufficiently large,

$$E\{U[Y(h, Z)]\} > U(y) - [U(y) - U(y')] = U(y') .$$

so that $Y(h, Z)$ is at least as good as the certainty of y' . Therefore, an infinite scale with correspondingly infinite information is optimal; more precisely, we can find a sequence of joint choices of scale and information, in the ratio, $2c/v$, which are increasingly preferred and preferred to any other given alternative for choices sufficiently far out on the sequence.

We need then a useful sufficient condition for the second integral in (3.13) to approach zero. Note that the upper limit of intergration is approaching negative infinity. For $z < 0$, $Y(h, z)$ is a convex function of h and therefore has a minimum defined by the first-order condition, $\partial Y / \partial h = 0$.

$$\partial Y / \partial h = c + (czh^{-1/2}/v) = 0 ,$$

so that,

$$h^{1/2} = -(z/v) .$$

Substitution into (3.11) yields,

$$\underline{Y}(z) = \min_h Y(h, z) = -cz^2/v^2 .$$

By construction,

$$(3.14) \quad Y(h, z) > \underline{Y}(z) , \text{ all } z < 0 .$$

Suppose that,

$$(3.15) \quad \int_{-\infty}^0 U[\underline{Y}(z)] \phi(z) dz > -\infty .$$

That is, the integral converges. Then, since $\lim_{h \rightarrow \infty} z^0(h, y) = -\infty$, it follows that,

$$(3.16) \quad \lim_{h \rightarrow \infty} \int_{-\infty}^{z^0(h, y)} U[\underline{Y}(z)] \phi(z) dz = 0 .$$

From the definition of $z^0(h, y)$ in (3.12) and from (3.14),

$$y > Y(h, z) > \underline{Y}(z) , \text{ for } z < z^0(h, y) < 0 .$$

Hence,

$$U(y) > U[Y(h, z)] > U[\underline{Y}(z)] \text{ in the same range} .$$

Multiply through by $\phi(z)$, and integrate from negative infinity to $z^0(h, y)$.

$$U(y) P[Z < z^0(h, y)] > \int_{-\infty}^{z^0(h, y)} U[Y(h, z)] \phi(z) dz$$

$$> \int_{-\infty}^{z^0(h, y)} U[\underline{Y}(z)] \phi(z) dz .$$

But the two extreme terms both approach zero, by Theorem 3 and (3.16), respectively.

Theorem 4. Under the hypotheses of Theorem 2, together with the condition,

$$\int_{-\infty}^0 U[-(c/v^2) z^2] \phi(z) dz > -\infty ,$$

any pair of decisions, a, h of scale and information respectively, is inferior in the sense of expected utility to some pair (a', h') with $a' = (2c/v) h'$ and h' sufficiently large.

Comment: If the hypotheses of Theorem 4 hold for all $v = 0$, there can be no competitive equilibrium with $v > 0$. If in addition it is assumed that the firm is risk-averse, then clearly with $v < 0$ the optimal choice of scale is 0 for any information. Hence, there can be no competitive equilibrium. Note that, as remarked in Section 1, the mean return, v , reflects market price and can be thought of as adjusting to produce equilibrium if possible. Hence, the only possibility for existence of equilibrium is that the condition of Theorem 4 fails to hold for some $v > 0$. Note that,

$$-(c/v^2) z^2 ,$$

is an increasing function of v for fixed c, z ; hence, it is possible that by decreasing v , the integral may be made to diverge for some $v > 0$. We explore some examples in the next section.

3. Examples of Existence and Non-Existence.

First, consider the case where U is bounded from below. In that case, the integral in the condition of Theorem 4 converges for any v . Therefore, there can be no competitive equilibrium with $v > 0$. In this case, however, the utility function cannot be concave, so it is conceivable that competitive equilibria exist with $v < 0$. It can however be shown that if $v < 0$ there cannot be optimal policy of the firm with both a and h finite positive numbers. Indeed, this follows directly from the first-order conditions preceding (3.10). If $v < 0$, then $E[U'(Y) Z] > 0$ from the first-order condition on a ; but then,

$$E[U'(Y)(\partial Y / \partial h)] < -c E[U'(Y)] < 0,$$

so that the first-order conditions cannot hold for both variables simultaneously.

Theorem 5. Under the conditions of Theorem 2, the optimal policy cannot have both a and h positive if $v < 0$.

This leaves open the question whether it is possible to have an equilibrium with $v < 0$ (non-positive expected profit) and no purchase of information. It is possible to find a utility function bounded from

below in which no equilibrium exists for any value of v , specifically, when $U(Y)$ is the cumulative standard normal distribution.

A case which leads to a more definite result is that in which U is concave with finite marginal utility at negative infinity. Under concavity, of course, there will be zero output for $v < 0$. For U concave, we have the inequality,

$$U(x) < U(y) + U'(y)(x - y) ,$$

for all x and y . Set $x = 0$, so that $U(0) < U(y) - U'(y)y$, for all y . Since $U'(y)$ is decreasing, $U'(y) < U'(-\infty)$ for all y . For $y < 0$, $U'(y)y > U'(-\infty)y$. Therefore,

$$U(y) > U(0) + U'(-\infty)y \text{ for } y < 0 .$$

Let $y = - (c/v^2) z^2$. Then,

$$\int_{-\infty}^0 U[-(c/v^2) z^2] \phi(z) dz > U(0) P(Z < 0) \\ - (c/v^2) U'(-\infty) \int_{-\infty}^0 z^2 \phi(z) dz .$$

Since $\sigma_z^2 = \int_{-\infty}^0 z^2 dz = 1$, it must be that $\int_{-\infty}^0 z^2 \phi(z) dz$ is finite. Therefore, the condition of Theorem 4 is satisfied.

Corollary to Theorem 4. Under the hypotheses of Theorem 2, if U is concave and $\lim_{y \rightarrow -\infty} U'(y)$ is finite, there is no non-trivial competitive equilibrium.

For another example with different implications, consider the exponential utility function,

$$(3.17) \quad U(y) = -r^{-1} e^{-ry} .$$

This utility function, with constant absolute risk aversion, has the advantage of yielding easily computable results, especially when combined with the assumption that the random variables are normally distributed. Substitute (3.17) into the condition of Theorem 4, together with,

$$\phi(z) = (2\pi)^{-1/2} e^{-z^2/2} .$$

The condition is then equivalent to,

$$\int_{-\infty}^0 e^{(c/v^2)rz^2 - z^2/2} dz < +\infty .$$

which holds if and only if $(c/v^2)r < 1/2$. This result suggests the possibility of competitive equilibrium if $(c/v^2)r > 1/2$, i.e., if $v < (2cr)^{1/2}$. Indeed, a closer examination shows that this is so.

Since Y is a linear function of Z , Y is normally distributed for any choice of a, h . Straightforward integration yields the well-known result,

$$E[U(Y)] \approx -r^{-1} e^{-rV},$$

where,

$$(3.18) \quad V = E(Y) - (r/2) \sigma_Y^2 .$$

Therefore, maximization of $E[U(Y)]$ under any constraints is equivalent to maximizing (3.18) under the same conditions. Here, we are maximizing

with respect to a and h . The general argument leading to Theorem 2 is valid here, so Y is given by (3.11). We seek to choose h to maximize V . From (3.11)

$$E(Y) = ch ,$$

$$\sigma_Y^2 = (2c/v)^2 h ,$$

so that,

$$v = [c - (2c^2 r/v^2) h] .$$

The optimization is now exactly parallel to that under constant returns to scale. If the expression in brackets is positive, the optimal decision is an infinitely large h and corresponding value of a , by (3.10); if negative, then a zero choice of both is optimal. But if the bracket is zero, then the firm is indifferent among all choices of a, h on the ray defined by (3.10).

Example 1. Under the assumptions of Theorem 2, if

$$U(y) = - r^{-1} e^{-ry} \text{ and } Z \text{ is normal,}$$

then there exists a competitive equilibrium with $v = (2cr)^{1/2}$. Any given firm can choose any scale, but the amount of information purchased is related to the chosen scale by the relation, $a = (2c/r)^{1/2} h$.

The last relation is derived by substituting the equilibrium value of v into (3.10).

Thus, on the one hand, no equilibrium can exist if the limiting marginal utility is finite, and on the other equilibrium does exist if the utility function is exponential and the distribution of the uncertainty is normal. A final example will clarify matters and suggest that the exponential represents in some sense a boundary case (when the random elements are normal). Let,

$$U(y) = -e^{(-y)^{1/2}} \quad \text{for } y \text{ sufficiently large negatively,}$$

and assume Z normal. Call this function, $\underline{U}(y)$. It is defined for $y > 0$, increasing where defined, and concave for $y < -1$. To find a utility function which is everywhere concave and increasing, we choose any $y_0 < -1$ and a suitable increasing concave function \bar{U} defined for $y > y_0$ agreeing with \underline{U} at y_0 in its value and in the values of its first two derivatives. For example, choose,

$$\bar{U}(y) = A \ln(y - y_1) + B ,$$

for suitably chosen values of the parameters A , B , and y_1 . Choose y_1 so that,

$$y_0 - y_1 = -\underline{U}'(y_0)/\underline{U}''(y_0) ;$$

then $y_1 < y_0$. For this value of y_1 , we can easily choose A and B so that $\bar{U}(y_0) = \underline{U}(y_0)$ and $\bar{U}'(y_0) = \underline{U}'(y_0)$. Because of the choice of y_1 it is easily seen that $\bar{U}''(y_0)/\bar{U}'(y_0) = \underline{U}''(y_0)/\underline{U}'(y_0)$ and therefore $\bar{U}''(y_0) = \underline{U}''(y_0)$. Hence if we define,

$$U(y) = \underline{U}(y) \text{ for } y = y_0 ,$$

$$= \bar{U}(y) \text{ for } y = y_0 ,$$

U is defined, strictly increasing and concave for all y . Now consider the condition of Theorem 4. The convergence of the integral depends only on \underline{U} . It is easy to see that the integral converges for all values of $v > 0$. Hence, no competitive equilibrium can exist.

4. Summary

The relation between competitive equilibrium and the possibility of technical information is more complex than I anticipated. Clearly, competitive equilibrium is not impossible, though its existence prescribes close relation between scale of output and amount of information purchased. On the other hand, there seem no simple conditions which would characterize the existence of equilibrium.

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